# What is numerical simulation? And why should I care?

KYOTO UNIV GRAD SCHOOL OF ENERGY SCIENCE

ENERGY CONVERSION SCIENCE DEPT.

Master's student

Yusuke Ikemi

# Self Introduction

- Name : Yusuke Ikemi
- Lab : Combustion and power engineering Lab
- Research : Heat transfer of impinging diesel spray
  (Large eddy simulation : LES)
- Hometown : Shiga Pref
- Hobby : Bakery hopping

# Today’s topic

- Terminology
- Why not just testing?
- Numerical simulations around us
- Dig down CFD(Computational Fluid Dynamics)

# Today’s topic

- Terminology
- Why not just testing?
- Numerical simulations around us
- Dig down CFD(Computational Fluid Dynamics)

# Definition of []

- Numerical Analysis

the study of algorithms that use numerical approximation for the problems of mathematical analysis

- Simulation

an approximate imitation of the operation of a process or system; that represents its operation over time

(often said as a training method)

- Numerical simulation

Calculation that is run on a computer following a program that implements a mathematical model for a physical system, whose models are too complex to provide analytical solutions.

# Today’s topic

- Terminology
- Why not just testing?
- Numerical simulations around us
- Dig down CFD(Computational Fluid Dynamics)

# Why not just testing?
# ----In some case, you cannot test!!

For example… Testing

- Nuclear explosion

  Dangerous, harmful

- Weather forecasting, earthquake

  Too huge,  you cannot make it happen

- Solar system

  Too slow to observe

- Chemical reactions

  Too fast and tiny to observe

<u>With simulation…</u>

Possible and safe

Calc whole earth

Fast forwarding

Close up and slow replay

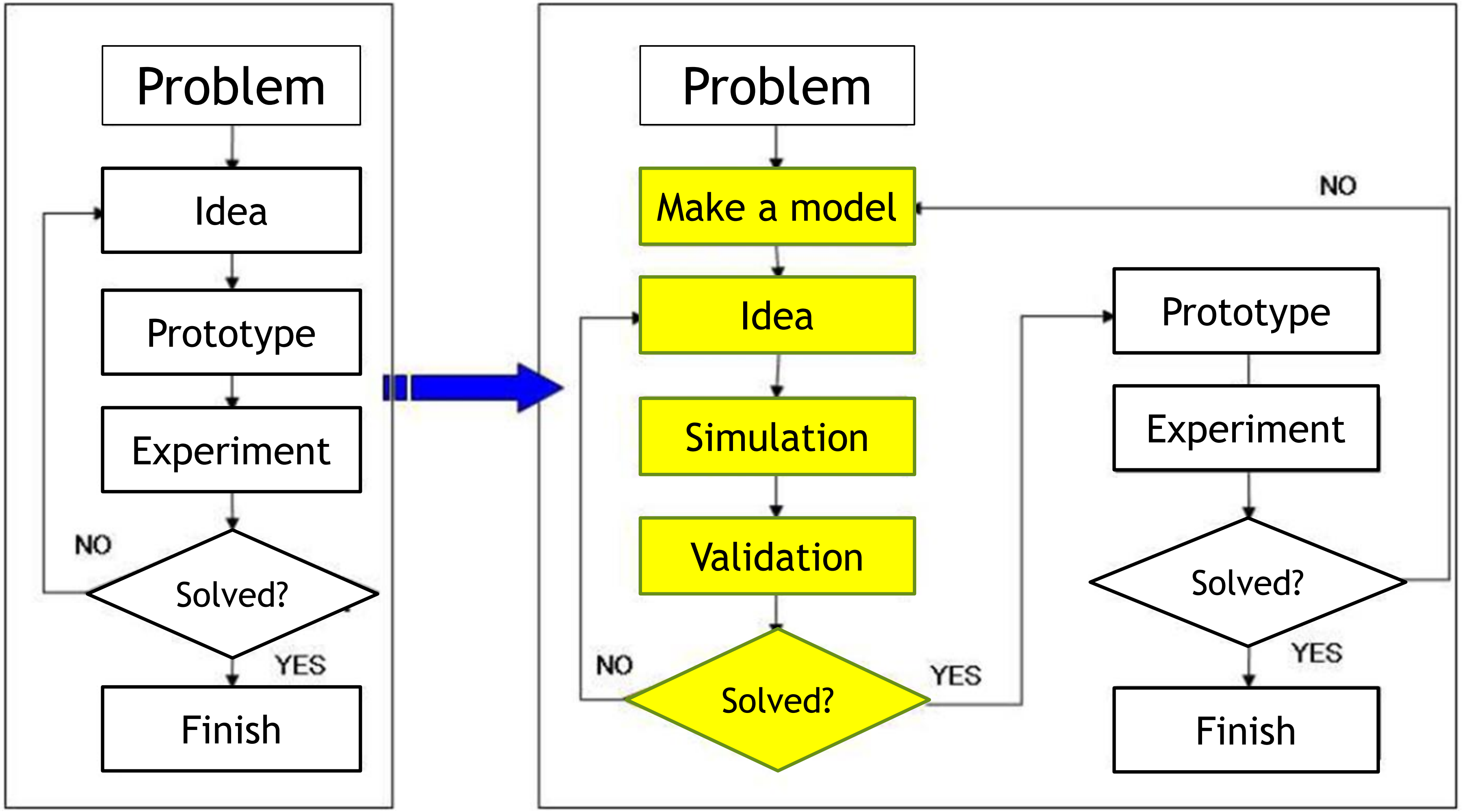
# Why not just testing? ----testing possible, but…..

Case1: you wanna build a new car!

| | Testing | Numerical Simulation |
|---|---|---|
| Costs | Material, Crafting,<br>Expensive Apparatus,,, | Machine resource<br>Software |
| Time | Build and analyze,<br>again and again | Machine time<br>(you can sleep) |
| Dangerousness | Flammables,<br>Chemicals,<br>Heavy stuff | No harm<br>(You can sleep) |
| Reproducibility | Setting Environment<br>costs a lot | Always you can control |
| Observability | Observation affects system<br>Limited information | Full information of system |

Numerical Simulation helps a lot!

# Why not just testing?




Save your time and cost
by decreasing experiments

# Today's topic

- Terminology
- Why not just testing?
- Numerical simulations around us
- Dig down CFD(Computational Fluid Dynamics)

https://en.wikipedia.org/wiki/Computer_simulation

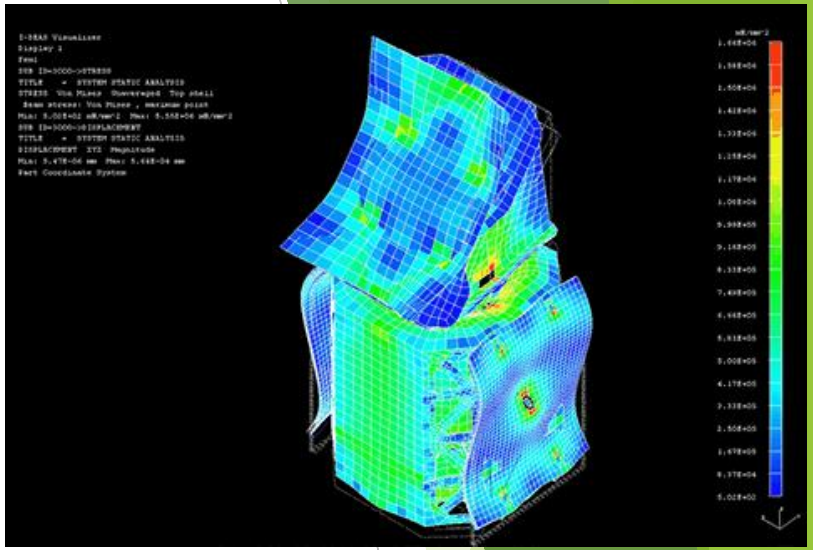
## Computer simulations are used in a wide variety of practical contexts, such as:

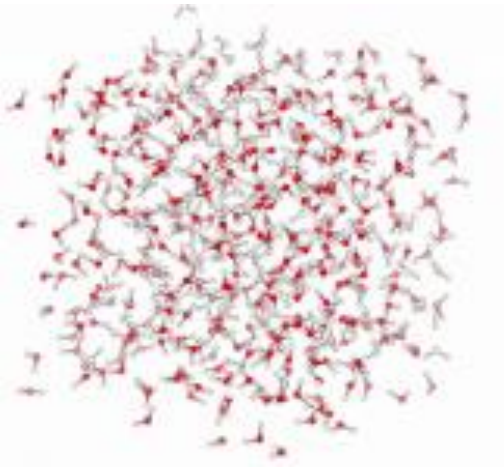
- analysis of air pollutant dispersion using atmospheric dispersion modeling
- design of complex systems such as aircraft and also logistics systems.
- design of noise barriers to effect roadway noise mitigation
- modeling of application performance[14]
- flight simulators to train pilots
- weather forecasting
- simulation of other computers is emulation.
- forecasting of prices on financial markets (for example Adaptive Modeler)
- behavior of structures (such as buildings and industrial parts) under stress and other conditions
- design of industrial processes, such as chemical processing plants
- strategic management and organizational studies
- reservoir simulation for the petroleum engineering to model the subsurface reservoir
- process engineering simulation tools.
- robot simulators for the design of robots and robot control algorithms
- urban simulation models that simulate dynamic patterns of urban development and responses to urban land use and transportation policies. See a more detailed article on Urban Environment Simulation.
- traffic engineering to plan or redesign parts of the street network from single junctions over cities to a national highway network to transportation system planning, design and operations. See a more detailed article on Simulation in Transportation.
- modeling car crashes to test safety mechanisms in new vehicle models.
- crop-soil systems in agriculture, via dedicated software frameworks (e.g. BioMA, OMS3, APSIM)

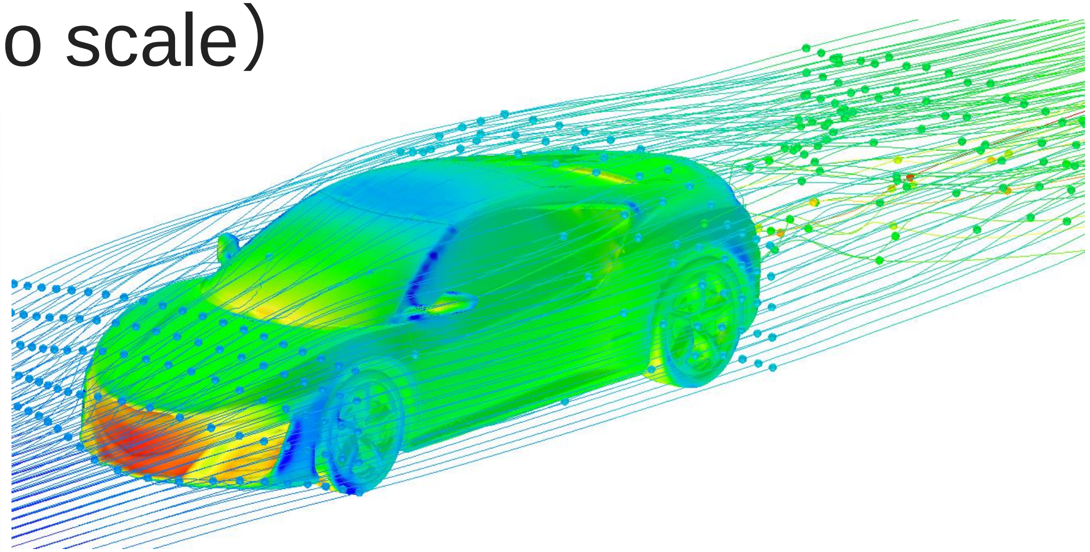
# Numerical simulations around us (Examples)

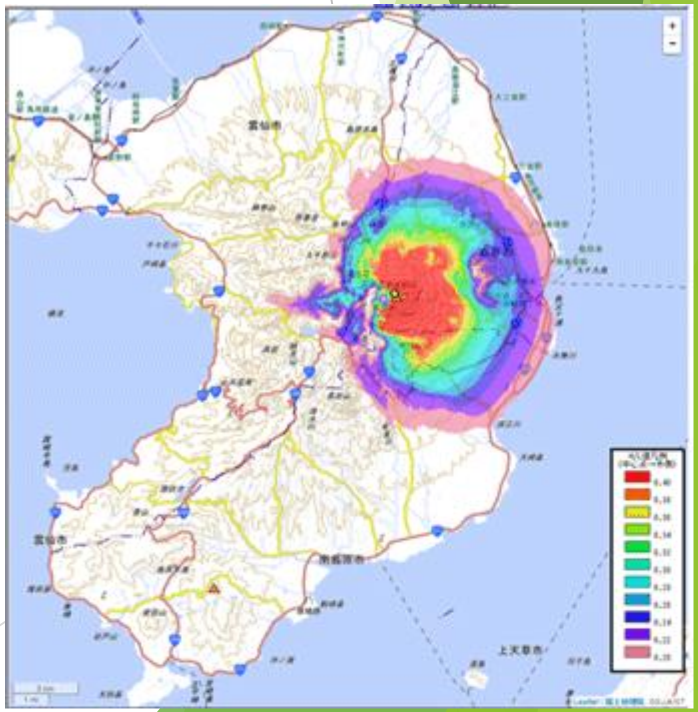
- Deep understanding of known
  （Natural Disaster）
- Optimization of scenalio
  （Decision, engineering processes）
- Prediction of unknown
  （Weather, nano scale）

Aero Space









Almost anything…

# Today’s topic

- Terminology
- Why not just testing?
- Numerical simulations around us
- Dig down CFD(Computational Fluid Dynamics) How it works inside a computer. As an example.

# Dig down CFD(Computational Fluid Dynamics)

- Computational Fluid Dynamics（CFD）

the simulation of fluids engineering systems using modeling and numerical methods

- Governing equation： Navier-Stokes

→a partial differential equation that describes the flow of viscous fluid substances.

The nonlinearity makes most problems difficult to solve analytically

Navier-Stokes

Pressure Gradient

$$\rho\frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}.\nabla)\vec{v} = -\nabla P + \vec{\gamma}\rho + \mu\nabla^2\vec{v}$$

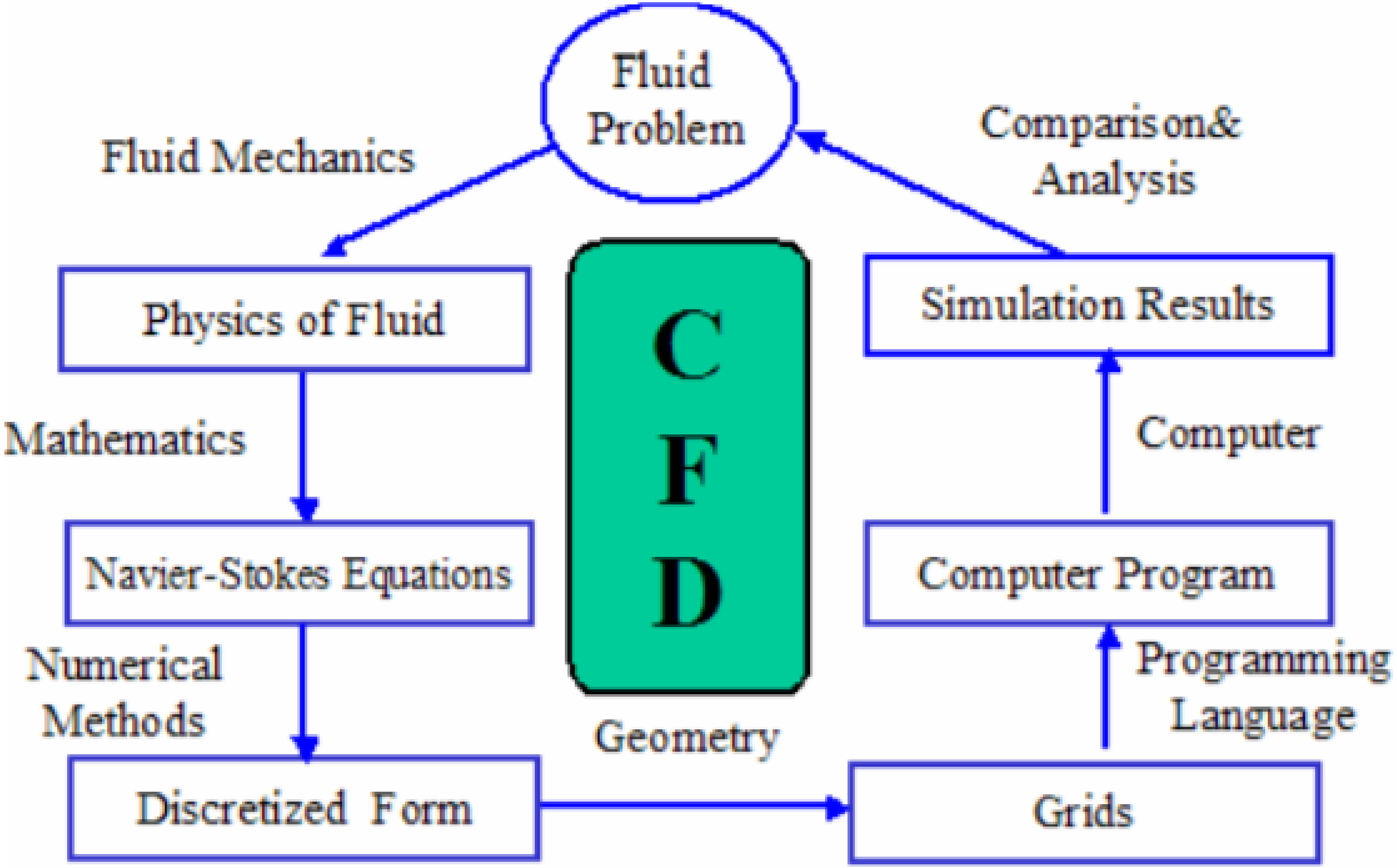
Local Acceleration

Convective Acceleration

Viscous term

Body force term

# Dig down CFD(Computational Fluid Dynamics)



# Dig down CFD(Computational Fluid Dynamics)

## Example

- Basic Equations

Divide calculation domain into tiny cells.

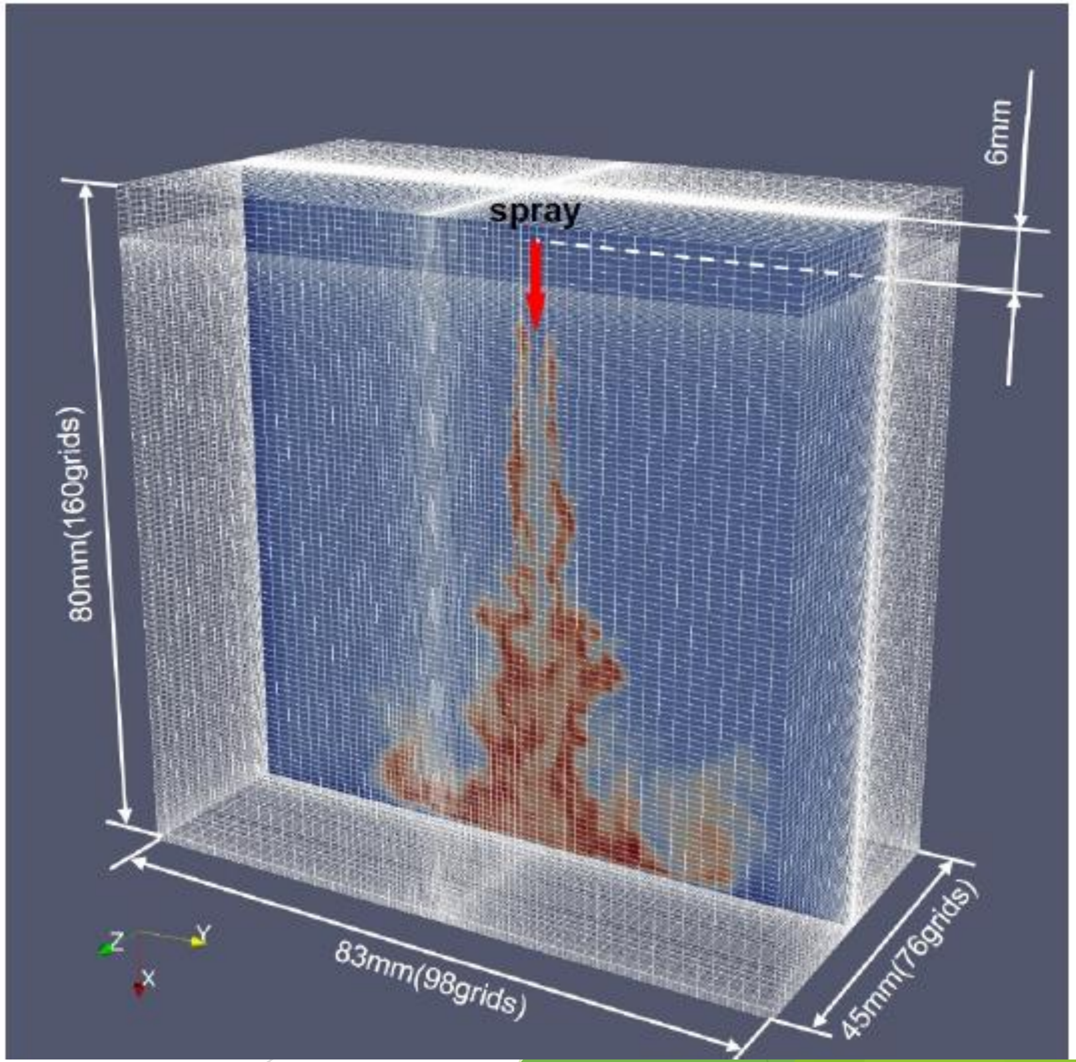
$$\frac{\partial \bar{\rho} V_f}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = \overline{\dot{\rho}^s} \quad , \qquad (1)$$

$$\frac{\partial \bar{\rho} \tilde{u}_i V_f}{\partial t} + \frac{\partial \left( \bar{\rho} \tilde{u}_i \tilde{u}_j + \tau_{ij}^{SGS} \right)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \overline{F_i^s} \, , \qquad (2)$$

$$\frac{\partial \bar{\rho} \tilde{f}_m V_f}{\partial t} + \frac{\partial \left( \bar{\rho} \tilde{f}_m \tilde{u}_j + j_{i,m}^{SGS} \right)}{\partial x_j} = -\frac{\partial}{\partial x_j} \bar{j}_{j,m} + \bar{\omega}_m + \overline{\dot{\rho}_m^s} \, , \qquad (3)$$

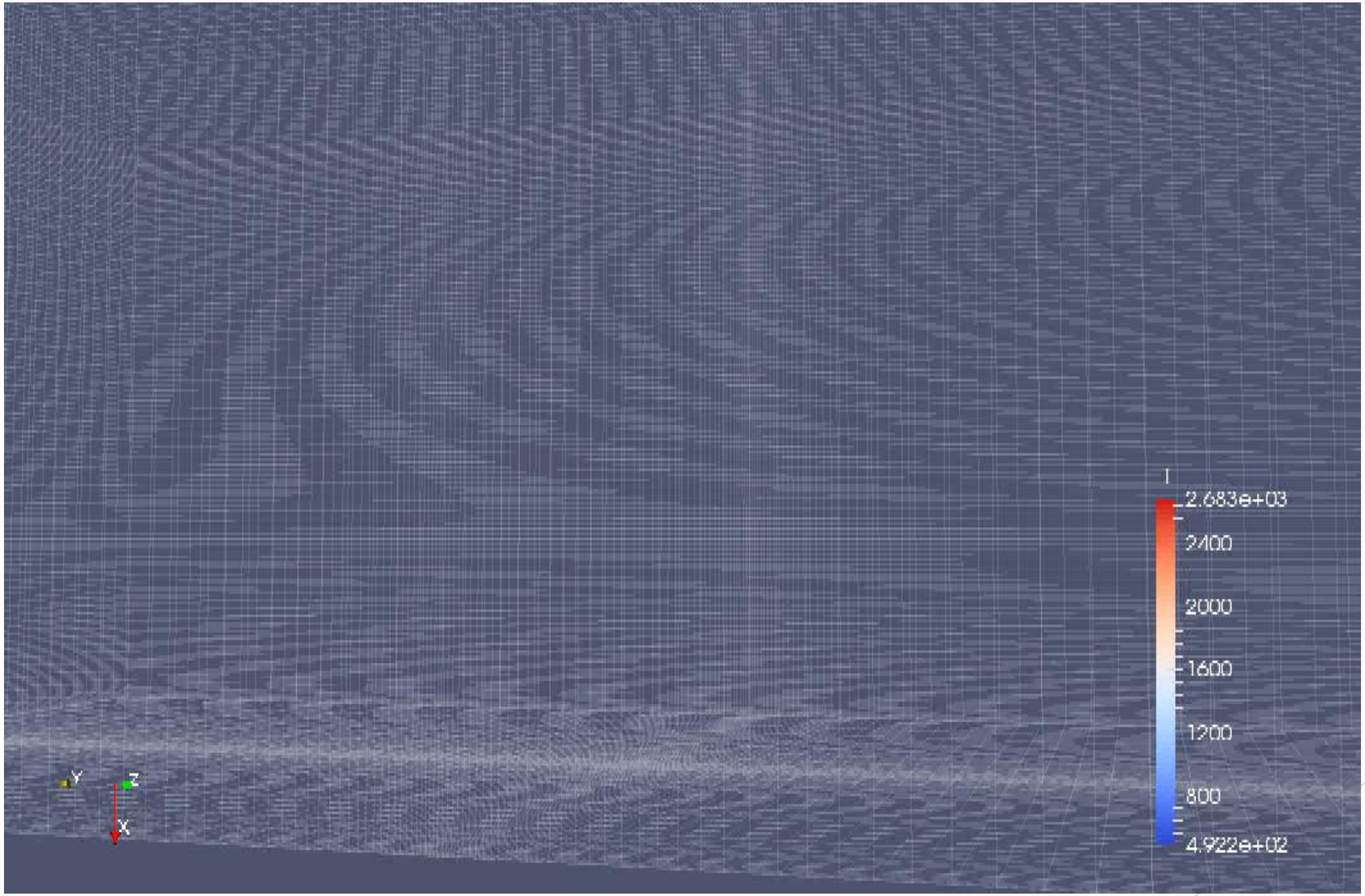
$$\frac{\partial \bar{\rho} \tilde{h} V_f}{\partial t} + \frac{\partial \left( \bar{\rho} \tilde{h} \tilde{u}_j + q_j^{SGS} \right)}{\partial x_j} = -\frac{\partial p_0 V_f}{\partial t} - \frac{\partial}{\partial x_j} \bar{q}_j + \overline{\dot{Q}^s} \, , \qquad (4)$$

About a half million cells



# Dig down CFD(Computational Fluid Dynamics)

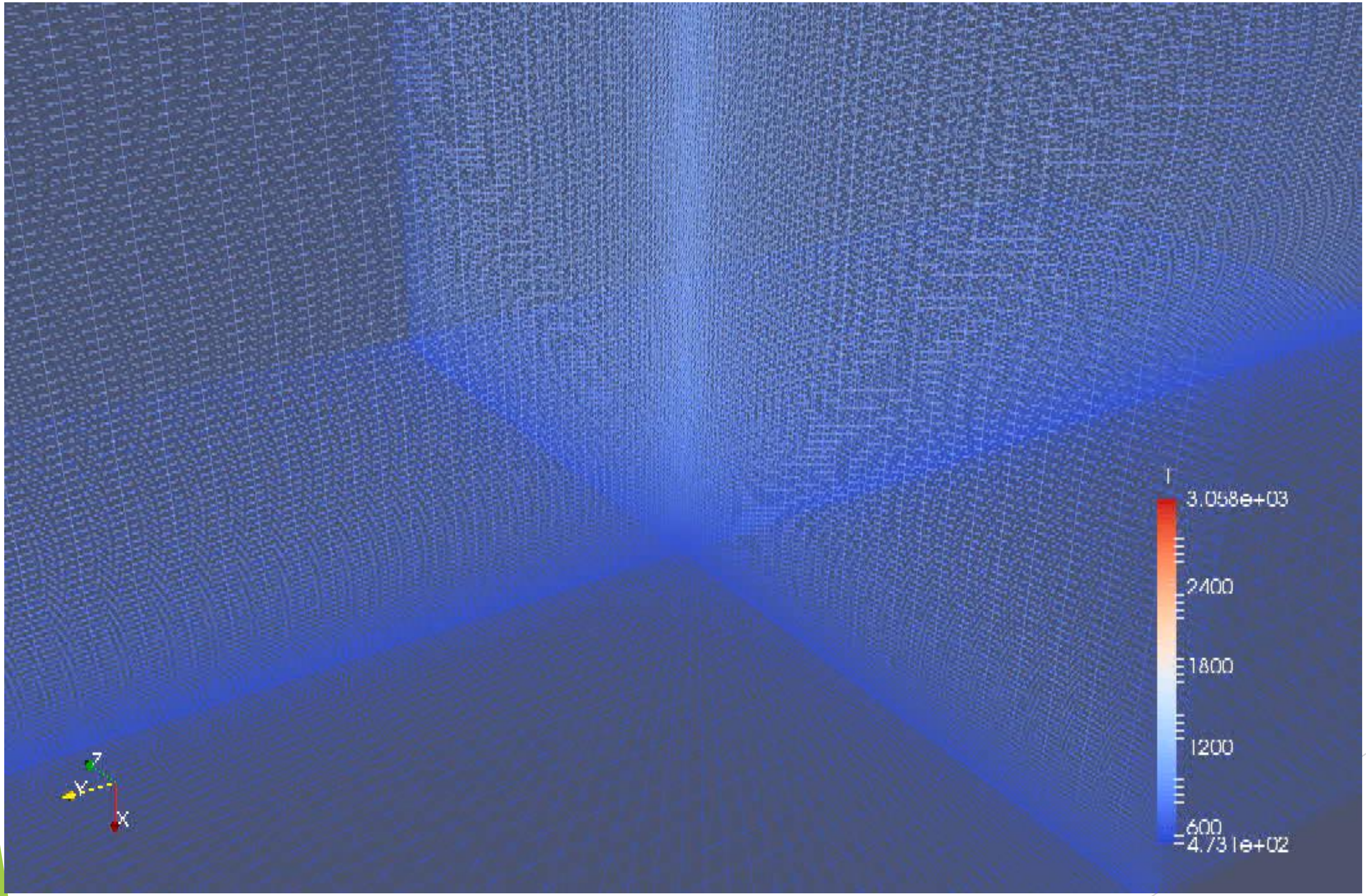
- Transient + Burning Flow




It took 6days
to calc
4ms combustion
With
Xeon 20 cores
workstation

# Dig down CFD(Computational Fluid Dynamics)

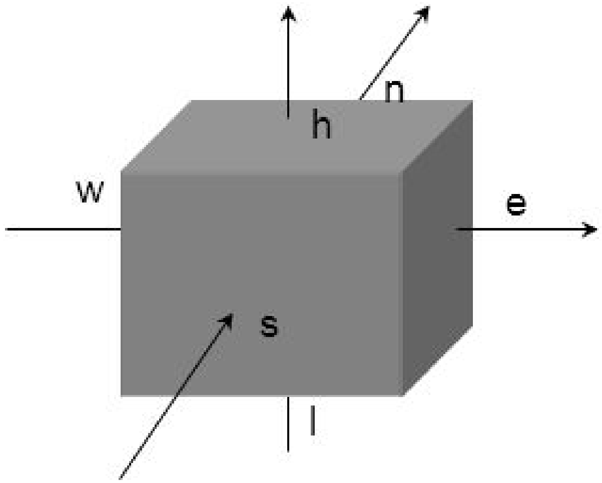
- Transient + Burning Flow



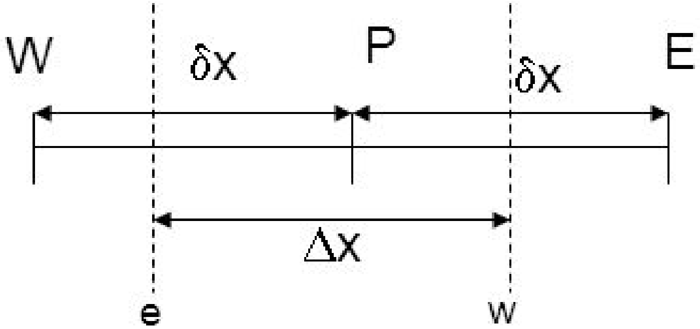
# Finite Volume Method（Discretization）

## - Conservation of $\phi$ for the finite volume

Divide the whole computation domain into sub-domains




One dimension:




Must Includ some error

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

- Finite volume is a *fixed* space in the flow domain with imaginary boundaries that allow the fluid to flow in and out.

- Integral conservation of the quantities such as mass, momentum and energy.

$\phi$

# Dig down CFD(Computatio

## Example

- Basic Equations

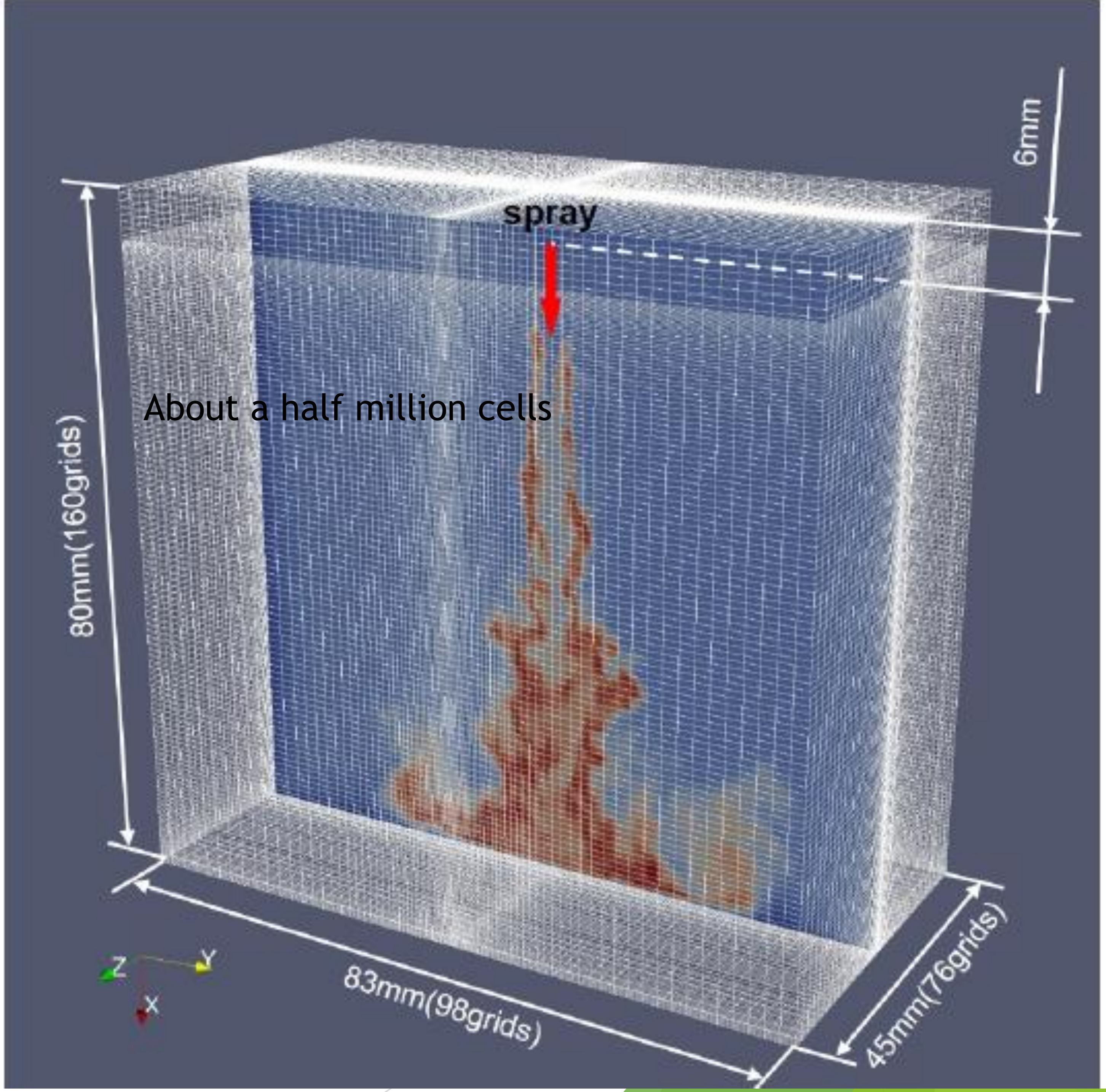
Divide calculation domain int
cells.

$$\frac{\partial \bar{\rho} V_f}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = \overline{\dot{\rho}^s},$$

$$\frac{\partial \bar{\rho} \tilde{u}_i V_f}{\partial t} + \frac{\partial\left(\bar{\rho} \tilde{u}_i \tilde{u}_j + \tau_{ij}^{SGS}\right)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \overline{F_i^s},$$

$$\frac{\partial \bar{\rho} \tilde{f}_m V_f}{\partial t} + \frac{\partial\left(\bar{\rho} \tilde{f}_m \tilde{u}_j + j_{i,m}^{SGS}\right)}{\partial x_j} = -\frac{\partial}{\partial x_j} \bar{j}_{j,m} + \bar{\omega}_m +$$

$$\frac{\partial \bar{\rho} \tilde{h} V_f}{\partial t} + \frac{\partial\left(\bar{\rho} \tilde{h} \tilde{u}_j + q_j^{SGS}\right)}{\partial x_j} = -\frac{\partial p_0 V_f}{\partial t} - \frac{\partial}{\partial x_j} \bar{q}_j +$$

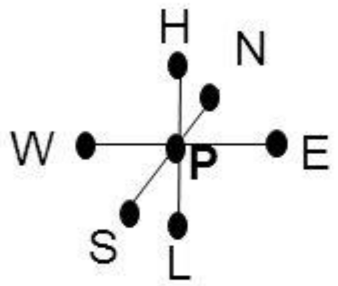


# 3D problem

Navier-Stokes

$$\rho\left[\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v+\frac{\partial u}{\partial z}w\right]=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

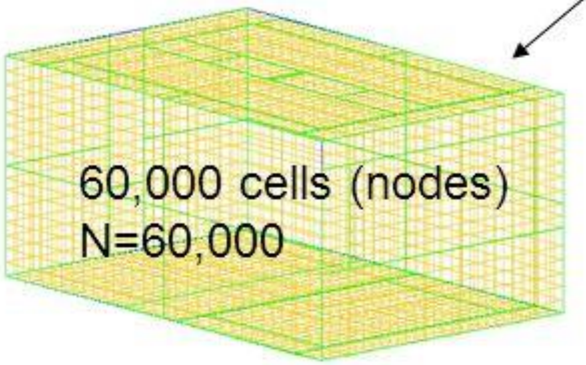
Discretization

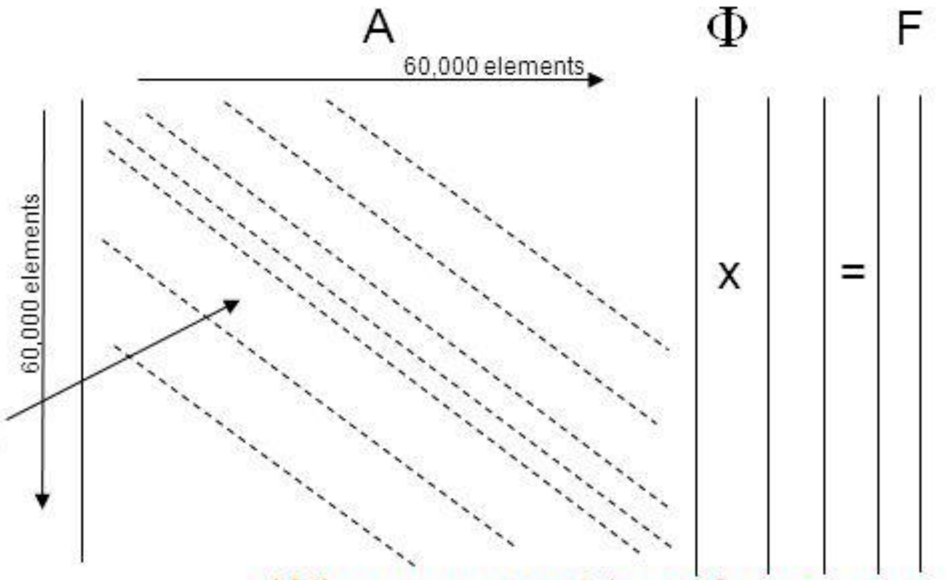



Equation in the general format:

$$a_P\Phi_P + a_E\Phi_E + a_W\Phi_W + a_S\Phi_S + a_N\Phi_N + a_H\Phi_H + a_L\Phi_L = f$$

Wright this equation for each discretization volume of your discretization domain




7-diagonal matrix

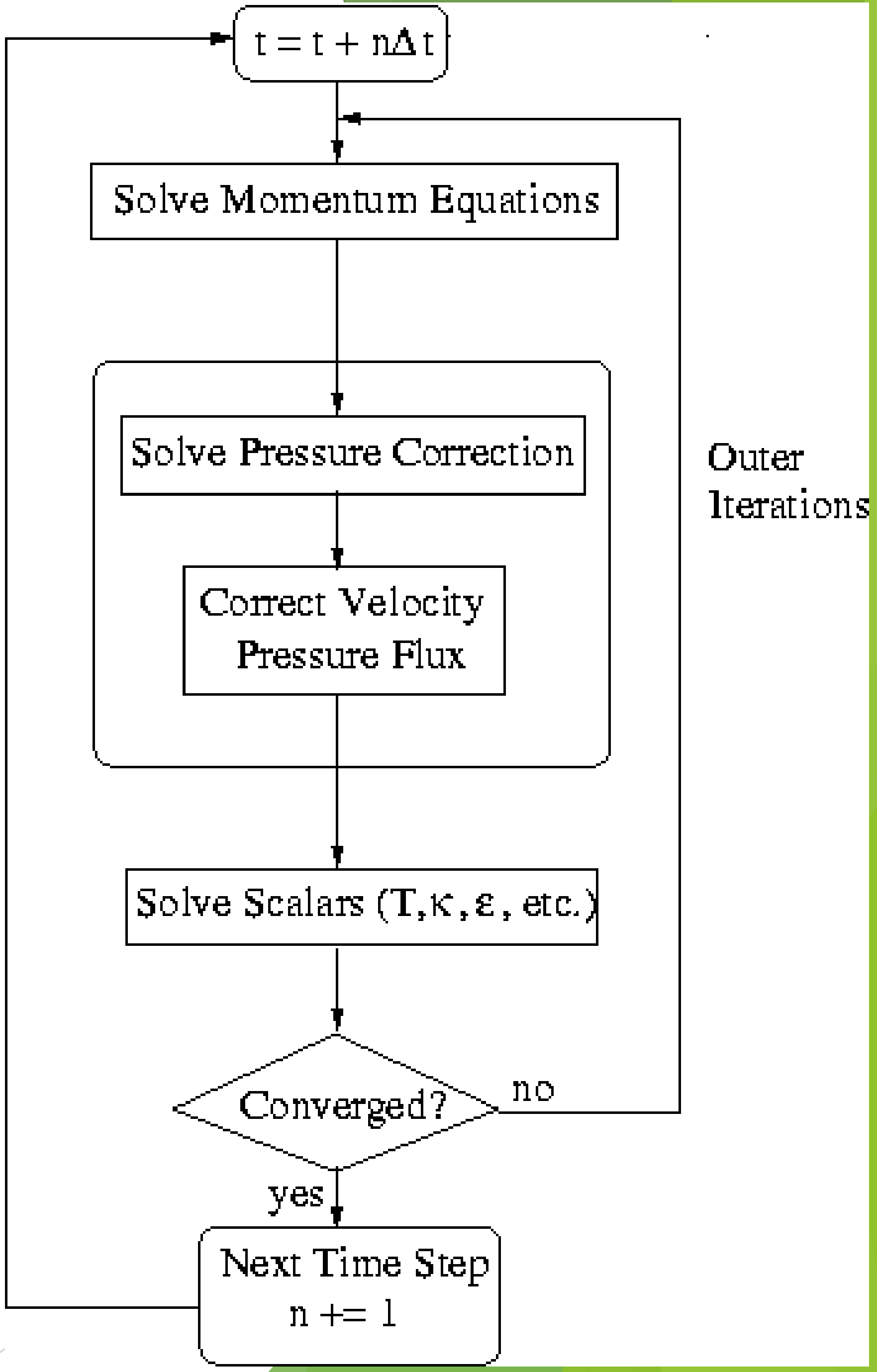
Just use Matrix-Solver

This is the system for only one variable ($\Phi$)

# CFD Flow

- Give initial condition.
- Solve conservation equation of all the cells.
- Check if its converged, if not, do it again
- If its converged (or if we get small enough error), go to next time step.
- Use last time variables as an initial condition.

$$N = \left[O\left(\frac{L}{l}\right)\right]^3 = O(Re^{9/4})$$



# Summary

- All [numerical simulation] has its [mathematical model] .

- Numerical simulation is regarded as important as testing.
  you must know how it works.

- There’s always a certain range of error in calculation.
  But experiment has also….

- Because of improvement of calc power, nowadays
complex NS is possible with low time cost.
It makes development faster!!

Thank you Any Question??